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Observation of large photonic band gaps and defect modes in one-dimensional networked waveguides

A Mir[1], A Akjouj[2,5], J O Vasseur[2], B Djafari-Rouhani[2], N Fettouhi[1], E H El Boudouti[3], L Dobrzynski[2] and J Zemmouri[4]

[1] Département de Physique, Faculté des Sciences, Université Moulay Ismail, Meknès, Morocco
[2] UMR CNRS 8024, UFR de Physique, Université de Lille I, 59655 Villeneuve d'Ascq, France
[3] Département de Physique, Faculté des Sciences, Université Mohamed I, Oujda, Morocco
[4] UMR CNRS 8523, UFR de Physique, Université de Lille I, 59655 Villeneuve d'Ascq, France

E-mail: Abdellatif.Akjouj@univ-lille1.fr

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Abstract
The photonic band structures and transmission spectra of serial loop structures (SLSs), made of loops pasted together with segments of finite length, are investigated experimentally and theoretically. These monomode structures, composed of one-dimensional dielectric materials, may exhibit large stop bands where the propagation of electromagnetic waves is forbidden. The width of these band gaps depends on the geometrical and compositional parameters of the structure and may be drastically increased in a tandem geometry made up of several successive SLSs which differ in their physical characteristics. These SLSs may have potential applications as ultrawide-band filters.

The propagation of electromagnetic waves in heterogeneous materials has received in recent years a great deal of attention. Of particular interest is the existence of photonic band gaps in the transmission spectra of artificial periodic structures with a spatially dependent dielectric constant. Various one-dimensional (1D), 2D, and 3D structures of these so-called *photonic crystals* have been studied [1–7]. In the forbidden bands, electromagnetic modes, spontaneous emission, and zero-point fluctuations are all absent [2]. The larger the band gaps, the more pronounced these properties are. In previous papers, we proposed [8, 9] a model of a 1D photonic crystal exhibiting very narrow pass bands separated by large forbidden bands. This model geometry is composed of an infinite 1D waveguide (*the backbone*) along which stars of N' finite side branches are grafted at N equidistant sites, N and N' being integers. The physical characteristics of this *star waveguide* are the periodicity, i.e. the distance between two sites, the length of each grafted branch, and the relative dielectric permittivity of the materials constituting the backbone and the side branches. The 1D nature of the model requires the two

[5] Author to whom any correspondence should be addressed.

characteristic lengths to be very much greater than the diameter of the backbone and the side branches. Thus only monomode propagation of electromagnetic waves needs to be considered in these networked waveguides. The stop bands originate from both the periodicity of the system and the states of the grafted branches which play the role of resonators. The width of the band gaps strongly depends on the contrast between the two characteristic lengths and the ratio between the two dielectric constants. Nevertheless, relatively large forbidden bands still remain when the two constituent materials are identical. This offers the possibility of photonic band-gap engineering in homogeneous materials, by tailoring their geometries.

In this paper, we propose a quasi-1D geometry, called *serial loop structure* (SLS), for a monomode networked waveguide. Such a structure may exhibit new features in comparison with star waveguides, such as the existence of larger gaps and the avoidance of the constraint on the boundary condition at the end of the side branches, which could be of potential interest for optical waveguide structures [10]. These features are essentially due to the loop structure, which is quite different from the case of a simple resonator [8, 9]. We report calculated band structures and transmission coefficients. We also show that the width of the band gaps may be enlarged by coupling several SLSs of different physical characteristics. Theoretical results are compared with transmission spectra measured with structures constituted of ordinary coaxial cables working in the frequency range of 1–500 MHz.

The 1D infinite SLS can be modelled as an infinite number of unit cells pasted together. In each unit cell, the two arms of the ring have different lengths d_2 and d_3. This results in a loop of length $d_2 + d_3$ (see figure 1(a)) which is pasted to a segment of length d_1. We focus in this paper on homogeneous SLSs where the media 1, 2, and 3 are made of the same material, characterized by its relative dielectric permittivity ε. The dispersion relation of the infinite SLS, which relates the angular frequency ω to the Bloch wavevector k, can be derived using the Green function method [11]. It can be written as $\cos(kd) = \eta(\omega)$ where d is the period of the structure and

$$\eta = \frac{1}{2\sin\left(\frac{\alpha L}{2}\right)\cos\left(\frac{\alpha\,\Delta L}{2}\right)}\left\{\sin(\alpha d_1)\left[\frac{5}{4}\cos(\alpha L) - \frac{1}{4}\cos(\alpha\,\Delta L) - 1\right] + \cos(\alpha d_1)\sin(\alpha L)\right\}. \tag{1}$$

Here $L = d_2 + d_3$, $\Delta L = d_2 - d_3$, $\alpha = \omega\sqrt{\epsilon}/c$, and c is the speed of light in vacuum.

Figure 1(b) displays the projected band structure (frequency $\omega/2\pi$ versus ΔL) for an infinite SLS for given values of L and d_1 such that $L = 1$ m and $d_1 = 0.5$ m. The shaded areas, corresponding to frequencies for which $|\eta| < 1$, represent bulk bands where waves are allowed to propagate in the structure. These areas are separated by minigaps where the wave propagation is prohibited. Inside these gaps, the dashed lines show the frequencies for which the denominator of η (equation (1)) vanishes: the dashed horizontal and curved lines, which correspond to the vanishing of $\sin(\alpha L/2)$ and $\cos(\alpha\Delta L/2)$ respectively [12], define the frequencies at which the transmission through a single loop becomes exactly equal to zero. In figure 1(b), one can distinguish two types of minigap: those of lozenge pattern that originate from the crossings of the zero-transmission lines; and the gaps around 100, 300, or 500 MHz (occurring for any value of ΔL) that are related to the periodicity of the structure. One interesting point to notice in the band structure of figure 1(b) is the fact that, at certain values of ΔL (for instance $\Delta L \sim 0.25$ m), one can obtain a series of narrow minibands separated by large gaps; this is because the points at which the minibands close align more or less vertically in such a way that a few successive bands may become very narrow.

We now turn to the study of the transmission probability. We start with a study of a simple example, namely a waveguide consisting of a simple asymmetric loop. The transmission

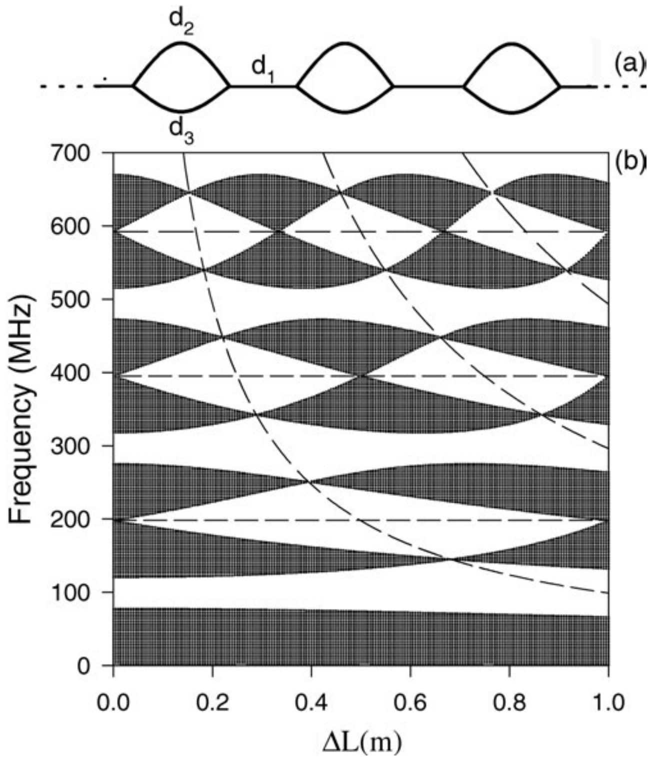

Figure 1. (a) A schematic diagram of the 1D SLS. The 1D media constituting the loop and the finite segments are assumed to be non-magnetic, isotropic, dielectric materials and are designated by an index i with $i = 1, 2$, or 3. The lengths of the three wires are denoted as d_1, d_2, and d_3 respectively. (b) The projected band structure of the SLS as a function of $\Delta L = d_2 - d_3$ for $d_1 = 0.5$ m and $L = d_2 + d_3 = 1$ m. The shaded areas represent the bulk bands. The dashed curves indicate the frequencies for which the denominator of η (equation (1)) vanishes.

coefficient T can be written as

$$T = \left| \frac{2(S_2 + S_3)S_2S_3}{(C_2S_3 + C_3S_2 + S_2S_3)^2 - (S_2 + S_3)^2} \right|^2 \tag{2}$$

where $C_i = \cosh(\alpha' d_i)$, $S_i = \sinh(\alpha' d_i)$, $\alpha' = \mathrm{j}\alpha$, and $\mathrm{j} = \sqrt{-1}$. The transmission is *equal to zero* only when $S_2 + S_3 = 0$, which leads to the vanishing of either $\sin(\frac{\alpha L}{2})$ or $\cos(\frac{\alpha\,\Delta L}{2})$. The zero-transmission frequencies, corresponding to the eigenmodes of a single loop, are thus given by

$$\omega_m = \frac{c}{\sqrt{\varepsilon}}(2m + 1)\frac{\pi}{\Delta L} \tag{3}$$

and

$$\omega_{m'} = \frac{c}{\sqrt{\varepsilon}}\frac{2m'\pi}{L}, \tag{4}$$

m and m' being integers.

In the particular case of a symmetric loop ($d_2 = d_3$, $\Delta L = 0$), the transmission coefficient becomes

$$T = \frac{16}{25 - 9\cos^2(\alpha' d_2)}. \tag{5}$$

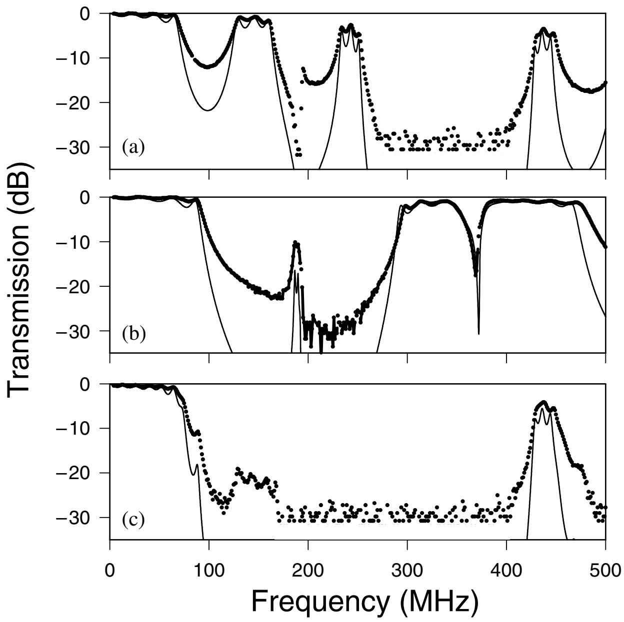

Figure 2. (a) Theoretical (solid curve) and experimental (dotted curve) variations of the transmission coefficient through the same structure (figure 1(b)), containing $N = 4$ loops, at $\Delta L = 0.29$ m. (b) As (a), but for $L = 0.5$ m and $\Delta L \approx 0.5$ m. (c) As (a), but for a tandem structure built up of the above SLSs, (a) and (b).

In contrast with the transmission coefficient of an asymmetric single loop, that of a symmetric one never reaches zero. The latter case is similar to that in double-barrier model systems. That is why, in symmetric SLSs, the gaps originate only from the periodicity. In contrast, in asymmetric SLSs, the gaps are due to the conjugate effect of the periodicity and the zero transmission associated with a single asymmetric loop, which plays the role of a resonator.

In the case where the number of asymmetric loops becomes greater than one, the zeros of transmission enlarge into gaps. The transmission coefficient (figure 2(a)) through a finite-size SLS containing $N = 4$ loops with $\Delta L = 0.29$ m clearly shows the existence of wide gaps separated by narrow bands especially at high frequencies. The theoretical transmission coefficient (solid curve) can be directly compared with experimental measurements (dotted curve). The experiment was performed using a tracking generator coupled to a spectrum analyser in the frequency range up to 500 MHz. All the constituents of the waveguide were standard 50 Ω coaxial cables assembled with metallic T-shaped connectors. Attenuation in the coaxial cables was simulated in the computations by a complex relative dielectric permittivity $\varepsilon = \varepsilon' - \mathrm{j}\varepsilon''$ where $\mathrm{j} = \sqrt{-1}$. The real part of ε is constant and equal to 2.3 while the imaginary part was obtained from the attenuation specification data specified by the manufacturer of the coaxial cables as a function of frequency f, i.e., $\varepsilon'' = 0.0146 f^{-0.47}$ with f in hertz. Despite the finite number of loops in figure 2(a), the transmission approaches −30 dB in regions corresponding to the observed gaps in the electromagnetic band structure of figure 1(b). The small peak observed at 190 MHz in the experiment (while it does not appear in the theory) arises because the actual length L of the loop is slightly different from the value of $L = 1$ m used in the calculation. As a consequence, there is a very narrow (almost flat) band in the dispersion curves of the actual structure at this frequency. In this respect, it is worth noticing

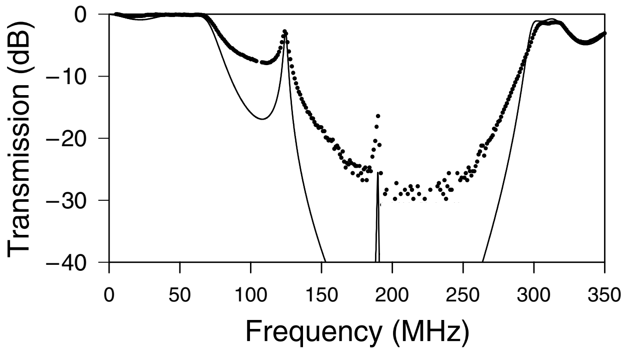

Figure 3. As figure 2(b), but with a defect segment of length $d_f = 0.13$ m in the middle of the structure.

that the general features discussed in figure 1(b) are valid for any values of d_1 and L and various values of ΔL. However, the shape of the band structure changes drastically for fixed values of d_1 and ΔL and various values of L.

Figure 2(b) shows the transmission spectrum for another different SLS with $d_1 = 0.5$ m and $\Delta L \approx L \approx 0.5$ m. This structure exhibits a large gap at frequencies lower than those in figure 2(a). The small feature appearing in the transmission spectrum around 180 MHz inside the large gap is associated with a flat band. It occurs because the actual length ΔL of the loops is slightly different from 0.5 m and the corresponding bands have a small width instead of being totally flat (see also figure 1(b) around the bulk band crossing points). Now, by associating the above SLSs in tandem, one obtains (figure 2(c)) an ultrawide gap where the transmission is cancelled over a large range of frequencies extending from 100 to 420 MHz. The width of this gap is measured at −30 dB. In this structure, the huge gap results from the superposition of the forbidden bands of the individual SLSs (figures 2(a), (b)). Theoretical and experimental results are in agreement within the experimental precision limits.

If a defect is included in the structure, a localized state can be created in the gap. A defect in a SLS can be realized by replacing a finite wire of length d_1 by a segment of length $d_f \neq d_1$ in one cell of the waveguide. Figure 3 shows the measured and calculated transmission spectrum for the structure depicted in figure 2(b) with a defect segment of length $d_f = 0.13$ m in the middle of this structure. The spectrum contains a peak at 120 MHz corresponding to the localized mode associated with the defect segment. The measured values are in remarkable agreement with theory. Let us emphasize that the frequency of the defect mode inside the gap depends on the length of the defect segment, whereas the intensity of the peak in the transmission spectrum depends on the number N of loops in the SLS and the localization of the defect segment. The band structure, like the defect modes, gives rise to well defined peaks in the delay time [13].

In conclusion, we have considered quasi-1D SLSs exhibiting very large photonic band gaps. Compared to those in other 1D systems such as star waveguides, the gaps observed in SLS may be significantly larger. In this study, the lengths of the finite wires were of the order of a metre and the gaps occurred in the hyperfrequency range. However, our theoretical model is universal and thus also valid for other frequency domains of the electromagnetic spectrum. For example, designing SLSs working at optical frequencies requires characteristic lengths d_1, d_2, and d_3 of the order of micrometres. The manufacturing of micrometric devices is now possible using high-resolution electron beam lithography [10]. Therefore, the fabrication of SLSs working at optical frequencies should now be feasible. Let us also stress that, for star waveguides, an important difficulty lies in the technical realization of the boundary condition at the free ends of the resonators, while this problem is avoided in SLSs.

Acknowledgments

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